

Key

Algebra 2
Notes 5-5 Function Operations

Obj: Perform algebraic operations on functions and composition.

Example 1.

Given $f(x) = 3x + 4$ and $g(x) = x^2 - 5x + 2$

Addition $(f+g)(x) = 3x+4+x^2-5x+2 = x^2-2x+6$

Subtraction $(f-g)(x) = 3x-4-(x^2-5x+2) = 3x-4-x^2+5x-2 = -x^2+8x-6$

What is the domain of the sum and difference.

$(-\infty, \infty)$

Example 2. The demand d , in units sold, for a company's new brand of cell phone at price x , in dollars, is $d(x)=5000-10x$. What is the company's expected revenue from cell phone sales in terms of the price, x ? Revenue = price x demand

$R = x(5000-10x)$
 $= 5000x - 10x^2$

What is the real world domain of this new function? $x \geq 0$

Example 3. Division. Let $f(x) = x-7$ and $g(x) = 2x^2 - 13x - 7$, find

$\left(\frac{f}{g}\right)(x) = \frac{x-7}{2x^2-13x-7} = \frac{x-7}{(2x+1)(x-7)} = \frac{1}{2x+1}$

**State the domain

$x \neq -\frac{1}{2}$
 $x \neq 7$

You try. Let $f(x) = x + 3$ and $g(x) = x^2 - 3x - 18$, find

$$\left(\frac{f}{g}\right)(x) = \frac{x+3}{x^2-3x-18}$$

$$= \frac{x+3}{(x+3)(x-6)}$$

$$= \frac{1}{x-6}$$

$$x \neq 6 \quad x \neq -3$$

$$(f \circ g)(x) = (x+3)(x^2-3x-18)$$

$$= \frac{x^3 - 3x^2 - 18x + 3x^2 - 9x - 54}{x^3 - 27x - 54}$$

Composite functions.

The composition of f with g , denoted $(f \circ g)(x)$, is defined by $f(g(x))$. That is, the output of $g(x)$ is the input of $f(x)$. *plug g into f*

Plug $g(x)$ into $f(x)$ for every x value.

Example 4. Evaluate the following. Given $f(x) = x^2$ and $g(x) = x + 1$

Find: $(f \circ g)(3)$

$$g(3) = 3 + 1 = 4$$

$$f(4) = 4^2 = 16$$

$(g \circ f)(3)$

$$f(3) = 9$$

$$g(9) = 9 + 1 = 10$$

You try. Let $f(x) = 4x^2 - 2x + 1$ and $g(x) = -5x$

Find: $(f \circ g)(2)$

$$g(2) = -10$$

$$f(-10) = 4(100) + 20 + 1 = 421$$

$(g \circ f)(2)$

$$f(2) = 4(4) - 2(2) + 1 = 13$$

$$g(13) = -5(13) = -65$$

$(f \circ f)(2)$

$$f(2) = 13$$

$$f(13) = 4(13)^2 - 2(13) + 1$$

$$= 4(169) - 26 + 1$$

$$= 676 - 26 + 1 = 651$$

Function Composition. Your answer will be a new function!

Given $f(x) = x^2$ and $g(x) = x + 1$

Find: $(f \circ g)(x)$

plug g into f

$$f(x+1) = (x+1)^2$$

$$= x^2 + 2x + 1$$

$(g \circ f)(x)$

plug f into g

$$g(x^2) = (x^2) + 1$$

$$= x^2 + 1$$

Example 5. Given $f(x) = \sqrt{x+7}$ and $g(x) = 2x - 5$

Find: $(f \circ g)(x)$

$(g \circ f)(x)$

$$\begin{aligned} f(2x-5) &= \sqrt{(2x-5)+7} \\ &= \sqrt{2x+2} \end{aligned}$$

$$g(\sqrt{x+7}) = 2(\sqrt{x+7}) - 5$$

Example 6. You have a \$10 gift certificate to a paint store. The store is offering 15% off your entire purchase of any paints and painting supplies. You decide to purchase a \$30 can of paint and \$25 worth of painting supplies.

\$55 total

Find the sale price of your purchase when the \$10 gift certificate is applied before the 15% discount.

$$55 - 10 = 45$$

$$45(.85) = 38.25$$

Find the sale price of your purchase when the 15% discount is applied before the \$10 gift certificate.

$$(55)(.85) = 46.75$$

$$\begin{array}{r} -10 \\ \hline 36.75 \end{array}$$

Write: functions for the discounts. Let x be the regular price, $f(x)$ be the price after the \$10 gift certificate is applied, and $g(x)$ be the price after the 15% discount is applied.

$$f(x) = x - 10$$

$$g(x) = .85(x)$$

Compose: the functions.

$$f(g(x)) = .85x - 10$$

%

then

sale

always

larger.

savings

$$\begin{aligned} g(f(x)) &= .85(x-10) \\ &= .85x - 8.5 \end{aligned}$$

↑

sale

the

%

